

"The Maximum Order of an Irreducible Covariant of a System of Binary Forms." By A. YOUNG, M.A., Clare College, Cambridge. Communicated by Major P. A. MACMAHON, D.Sc., F.R.S. Received September 26, 1903.

It has been suggested to me that an incidental result of a paper I have recently communicated to the London Mathematical Society may be of interest. In the paper in question it is proved that all covariants of a system of binary n -ics are linearly expressible in terms of—

(i) Covariants of the form

$$(a_1 a_2)^{\lambda_1} (a_2 a_3)^{\lambda_2} \dots (a_{\delta-1} a_{\delta})^{\lambda_{\delta-1}} a_{1x}^{n-\lambda_1} a_{2x}^{n-\lambda_1-\lambda_2} \dots a_{\delta x}^{n-\lambda_{\delta-1}},$$

where

$$\lambda_1 \leq 2^{\delta-2}, \quad \lambda_2 \leq 2^{\delta-3}, \quad \dots \quad \lambda_{\delta-1} \leq 1$$

(ii) Covariants having a symbolical factor

$$(ab)^{\lambda} (bc)^{n-\lambda} (ca)^{\rho}$$

(iii) Products of covariants.

Mr. J. H. Grace, in a note appended to the paper, has deduced from this result a means of calculating the maximum order, in the variables, of an irreducible covariant of a system of quantics. If no quantic of the system is of order exceeding n , the maximum order of an irreducible covariant is the greatest of the numbers

$$n\delta - 2^{\delta} + 2,$$

where δ is an integer,—in fact, the degree in the coefficients of the covariant in question.

If $n = 2^i + k$ where $k \not\geq 2^i$, it will be seen that the maximum is

$$(i+1)(2^i+k) - 2^{i+1} + 2.$$

The covariant of maximum order is then

$$(a_1 a_2)^{2^{i-1}} (a_2 a_3)^{2^{i-2}} \dots (a_{i-1} a_i)^2 (a_i a_{i+1}) a_{1x}^{n-2^{i-1}} \dots a_{ix}^{n-1}.$$

There are strong reasons for believing this covariant to be actually irreducible; in the contrary case a reduction is obtained for certain forms classed as perpetuants.*

* MacMahon, 'Camb. Phil. Trans.,' vol. 19, p. 234; Grace, 'Lond. Math. Soc. Proc.,' vol. 35, p. 107.

The following is a table giving the maximum order for all values of n from 1 to 100, together with the degree, δ , of the covariant of maximum order:—

n .	Max. order.	δ .	n .	Max. order.	δ .	n .	Max. order.	δ .	n .	Max. order.	δ .	n .	Max. order.	δ .
1	1	1	21	75	5	41	184	6	61	304	6	81	441	7
2	2	1, 2	22	80	5	42	190	6	62	310	6	82	448	7
3	4	2	23	85	5	43	196	6	63	316	6	83	455	7
4	6	2, 3	24	90	5	44	202	6	64	322	6, 7	84	462	7
5	9	3	25	95	5	45	208	6	65	329	7	85	469	7
6	12	3	26	100	5	46	214	6	66	336	7	86	476	7
7	15	3	27	105	5	47	220	6	67	343	7	87	483	7
8	18	3, 4	28	110	5	48	226	6	68	350	7	88	490	7
9	22	4	29	115	5	49	232	6	69	357	7	89	497	7
10	26	4	30	120	5	50	238	6	70	364	7	90	504	7
11	30	4	31	125	5	51	244	6	71	371	7	91	511	7
12	34	4	32	130	5, 6	52	250	6	72	378	7	92	518	7
13	38	4	33	136	6	53	256	6	73	385	7	93	525	7
14	42	4	34	142	6	54	262	6	74	392	7	94	532	7
15	46	4	35	148	6	55	268	6	75	399	7	95	539	7
16	50	4, 5	36	154	6	56	274	6	76	406	7	96	546	7
17	55	5	37	160	6	57	280	6	77	413	7	97	553	7
18	60	5	38	166	6	58	286	6	78	420	7	98	560	7
19	65	5	39	172	6	59	292	6	79	427	7	99	567	7
20	70	5	40	178	6	60	298	6	80	434	7	100	574	7